苏菲拉底 THE SOPHIE LAB

本期追问

无意义的原子如何堆叠出会说「我」的自我?

形式系统的符号，怎么会承载起意义?

为什么哥德尔、埃舍尔与巴赫能被编成同一条金带?

视频精读 VIDEO STUDY

MIT Godel Escher Bach Lecture 1

来源：YouTube 视频（https://www.youtube.com/watch?v=lWZ2Bz0tS-s）
节目发布：2012-12-02 · 全文 85 段 · 页边注释 63 条 · 生词词组 81 条

好视频，值得精读

目录 CONTENTS

第一部分 · 全文对照

第二部分 · 生词总表 · 复习用

第一部分　全文对照 · 附页边注释与生词标注

01　课程缘起与授课背景

0:00　Justin Curry

The following content is provided under a Creative Commons license. Your support will help MIT Open Courseware continue to offer high-quality educational resources for free. To make a donation or view additional materials from hundreds of MIT courses, visit MIT Open Courseware at ocw.mit.edu. Hi. Hello. Welcome to Gödel, Escher, Bach: An Eternal Golden Braid. my name is Justin Curry and I'm a senior in mathematics here at MIT. I've spent the last year at Cambridge University, UK. and the summer before that living in Germany. So it's kind of a reverse culture shock coming back, but I'm excited to teach Gödel, Escher, Bach again. I taught this course in spring 2006. It was a 10-week course then and we attempted the impossible task of trying to get through this thick monster all in one go. And it's impossible. most undergrads can't get through it in 13 weeks. I got through it in about 7 years. So you're going to be attempting a feat here, not to complete the entire book, but to get the essence of Gödel,

以下内容基于知识共享许可协议提供。您的支持将帮助 MIT 开放课程继续免费提供高质量的教育资源。如需捐赠或浏览来自数百门 MIT 课程的更多资料，请访问 MIT 开放课程网站 ocw.mit.edu。嗨。大家好。欢迎来到《哥德尔、埃舍尔、巴赫：集异璧之大成》这门课。嗯，我叫 Justin Curry，是 MIT 数学系的大四学生。过去一年我在英国剑桥大学，再之前那个夏天我住在德国。所以回来之后有点反向文化冲击，不过我很兴奋能再次讲授《哥德尔、埃舍尔、巴赫》。嗯，我在 2006 年春季学期教过这门课。那时是一门 10 周的课程，我们尝试了一件不可能的事——想一口气啃完这本厚得吓人的大部头。而这是不可能的。嗯，大多数本科生 13 周也读不完。我大概花了 7 年才读完。所以嗯，你们在这里要挑战的，并不是读完整本书，而是把《哥德尔、埃舍尔、巴赫》的精髓提炼出来。

《哥德尔、埃舍尔、巴赫》（GEB）是侯世达1979年出版的著作，1980年获普利策非虚构类奖，中译本名《集异璧》。本课是MIT开放课程面向高中生的研讨课。

reverse culture shock *phr.*
反向文化冲击（回到本国后反而不适应）

feat /fiːt/ *n.*
壮举，艰难的成就

1:07

Escher, Bach out. but I want to make sure we introduce everybody. just to get people's names. This will help me take attendance and it will also just I also want you to say what is it when you read the course catalog that interested you most and why essentially why you're sitting here today. I'm curious. so what is the idea behind this book? I interviewed a good many of you this morning and just to make sure that you guys felt comfortable with mathematics. This course isn't directly about mathematics.

嗯，不过我想确保大家先互相认识一下。嗯，就是想知道大家的名字。这能帮我点名，同时嗯，我也想请你们说说，你们在课程目录里读到什么内容最吸引你，以及为什么，嗯，本质上就是你今天为什么会坐在这里。嗯，我很好奇。嗯，那么这本书背后的核心想法是什么呢？嗯，今天早上我面谈了你们当中相当多的人，只是想确认你们对数学感到自在。这门课并不直接关于数学。

02 核心问题：自我从何而来

1:44

There's a lot of mathematics being talked about. Yes, do you have a question? What's the class about? Okay, so that's what I'm going to go through right now. the here is that Douglas Hofstadter is interested in one primary question. And that question is, how does a self come out of things which have no selves? How is it that all these carbon atoms and molecules and proteins which make us up in the physical universe, how do they go from being meaningless to developing into an entity which can refer to itself?

> 侯世达的核心问题：无意义的物质（原子、蛋白质）如何涌现出能自我指涉的「我」。这是全书主线，也是意识研究的中心难题。

> **entity** /ˈentəti/ *n.*
> 实体，独立存在物

嗯，课上会谈到很多数学。是的，你有问题吗？这门课到底讲什么？好的，那正是我现在要讲的。嗯，这里的关键是，Douglas Hofstadter 关注的是一个核心问题。那个问题就是：一个“自我”是如何从没有自我的东西中产生的？怎么会有这么多构成我们的碳原子、分子和蛋白质，在这个物理宇宙里，它们如何从毫无意义，发展成一个能够指涉自身的实体？

2:23

Like right now I'm saying I think this. I think you like this. I'm meeting all of you as individuals. Each one of you claim to have a self. You might remember Descartes' famous quote, I think therefore I am. So it seems like the I when I say the I, I mean the things we call ourselves as a real existent thing. But it's a complex question. How do we How do we get eyes out of non-eyes? And that's kind of going to be the goal over here. So I'm just going to call it I. But how do you get to an I? You get to an I by having a bunch of meaningless primitives.

比如现在我说“我认为如此”。我认为你喜欢这个。我正在把你们每个人当作独立的个体来认识。你们每一个人都声称拥有一个自我。你们可能记得笛卡尔那句名言：我思故我在。所以看起来，当我说“我”的时候，这个“我”指的是我们称之为“自己”的那个真实存在的东西。嗯，但这是个复杂的问题。我们如何——我们如何从“非我”中得到“我”？嗯，这大致就是我们这门课的目标所在。那么嗯，我就把它写作“我”。但你怎么才能得到一个“我”呢？你是通过一堆毫无意义的基本元素得到“我”的。

「我思故我在」出自笛卡尔《谈谈方法》(1637)，以思维活动本身作为不可怀疑的存在证明，是近代西方哲学的起点。

primitives /ˈprɪmətɪvz/ *n.*
基本元素，原始单元（逻辑/计算术语）

3:03

Things like atoms proteins, molecules, I should say if I want to etc. Like this is what you're made up of, but none of these things mean anything. None of these things have eyes or selves, but you do. So what's the What's the relationship here? Douglas Hofstadter he wrote this book back in the 70s when he was doing graduate school in physics. And this was after him doing math undergrad at Stanford. he believed that he saw the he saw the answer when he was playing around with mathematics in the very formal systems we play with. Like when we write down things like 2 + 2 = 4, these are just these are just symbols and as we go through the day I'll show you completely equivalent ways of doing addition which will look like this and these are just logical primitives like and if you've seen any set theory and you know don't feel scared if you haven't seen any of these symbols but like there exists an X for every we give these interpretations but the idea is that mathematics can be reduced

像是原子、蛋白质、分子，如果我想的话还可以写等等。就像这些是构成你的东西，但这些东西本身没有任何任何东西。这些东西都没有眼睛，也没有自我，但是——但是你有。那么这中间的关系是什么呢？嗯，侯世达（Douglas Hofstadter），他在七十年代读物理研究生的时候写了这本书。在那之前他在斯坦福读的数学本科。嗯，他相信自己看到了——他在摆弄数学、摆弄我们所用的那些非常形式化的系统时，看到了答案。比如当我们写下 2 + 2 = 4 这样的东西时，这些只不过是——这些只不过是符号。而且随着今天课程的推进，我会给你们展示一些完全等价的做加法的方式，看起来会是这个样子，嗯，而这些只是逻辑上的基本单元，比如——如果你学过一点集合论的话，你就知道；如果你从没见过这些符号也别害怕，比如「存在一个 X」「对于每一个」，我们给它们赋予解释，但核心想法是：数学可以被还原成

> 侯世达在斯坦福读数学本科，后在俄勒冈大学攻读物理学博士（其成果「侯世达蝴蝶」是能级谱分形图），GEB写于研究生时期。

4:17

To a bunch of meaningless operations just symbol shunting but what's interesting is that within mathematics there exists a an equivalent to a self-reference. This is this is this is a bunch of atoms and proteins referring to itself calling itself an I. what happens here and this is this is going to be kind of underneath the name of Gödel

一堆毫无意义的操作，就是符号的搬来搬去。嗯，但有意思的是，在数学内部存在着一种与自指等价的东西。这（指人）是一堆原子和蛋白质在指称它自己，把自己叫做「我」。嗯，这里发生了什么呢——而这部分大体上会挂在哥德尔的名下——

> 「符号搬弄」（symbol shunting）指形式系统中只按规则移动符号、不涉及含义的操作。数学中的自指与人的自我意识被类比为同一现象。

shunting /ˈʃʌntɪŋ/ *n.*
搬移，调动（symbol shunting 指机械地搬弄符号）

self-reference /ˌselfˈrefərəns/ *n.*
自指，自我指涉

4:49

Is we're going to get to some incompleteness theorems we're going to get to some statements which in mathematics refer to themselves and the question of how this happens we understand this rigorously. Mathematicians have worked out how do we go from meaningless symbols to something which refers to itself and which has meaning. The claim then is that these two systems are equivalent and this is really the profound idea. I'm going to draw this symbol and I'm going to use a term called isomorphism and isomorphism is basically an equals to an equals in a different sense but the idea here is in many ways we can link atoms and proteins to kind of logical symbolic primitives in mathematics and we understand how we get self-reference in mathematics so maybe we can use this to understand how we get I's how self comes out of non-self.

我们会讲到一些不完备性定理，会讲到数学中一些指称自身的陈述，而这是如何发生的，这个问题我们是有严格理解的。数学家们已经搞清楚了：我们如何从毫无意义的符号，走到某种指称自身、并且有意义的东西。那么这里的主张就是：这两个系统是等价的，而这才是真正深刻的想法。我要画下这个符号，我要用一个词叫「同构」（isomorphism）。同构基本上就是另一种意义上的等号，但这里的想法是，在很多方面我们可以把原子和蛋白质，跟数学里那些逻辑符号层面的基本单元联系起来。而我们理解数学中自指是怎么产生的，所以也许我们可以借此来理解「我」是怎么来的，自我是怎么从非自我中产生的。

哥德尔1931年发表不完备性定理：任何包含算术的一致形式系统中都存在指涉自身、为真却不可证明的命题。这是全书的数学支柱。

incompleteness /ˌɪnkəmˈpliːtnəs/ *n.*
不完备性（哥德尔不完备性定理）

rigorously /ˈrɪɡərəsli/ *adv.*
严格地，严密地

isomorphism /ˌaɪsəˈmɔːrfɪzəm/ *n.*
同构（结构间保持对应关系的映射）

5:42

This is a really tall order but we're to try to do it and that's what this book attempts to do. And what I've done is isolate the chapters in this book, which I think are most pertinent to pertinent to this string of thought. But basically what we're going to do is we're going to learn how it works in mathematics. We're going to go from logical primitives and work up to self-reference and talk about Zen Buddhism, consciousness, etc. But that's going to happen as we leap over here, cuz we're going to we're going to work up, down, and then around. And we'll conclude the course with some interesting questions about artificial intelligence and how intelligent things come out of unintelligent things.

全课路线图：从逻辑基本元素搭建到自指，再延伸到禅宗、意识与人工智能——即「智能如何从无智能的东西中产生」。

tall order *phr.*
艰巨的任务，过高的要求

pertinent /ˈpɜːrtnənt/ *adj.*
密切相关的，切题的

这是个非常艰巨的任务，但我们要试着去做，而这也正是这本书试图做的事。我做的事情是，把这本书里我认为跟这条思路最相关的章节挑了出来。但基本上我们要做的是：先学会它在数学里是怎么运作的。我们会从逻辑基本单元出发，一路搭建到自指，然后聊禅宗、意识等等。但那要等我们跳到这边才会发生，因为我们要往上走、往下走，然后绕一圈。课程最后我们会以一些关于人工智能的有趣问题收尾：有智能的东西是怎么从没有智能的东西里产生的。

03 五大思维工具总览

6:21

So, when I was teaching this course 2 years ago, or two springs ago, I ran into kind of five things, which I viewed as really important tools for thinking. And this is kind of I've had to condense a little bit into my famous tools for thinking lecture. The idea here is that Gödel, Escher, Bach has an incredible number of conceptual tools for thinking about this complex problem of how do you go from a non-self to a self? And just outline these real quick. we're going to have

五个思维工具是理解GEB的脚手架：同构、递归、悖论、无穷、形式系统。本讲按此顺序展开。

condense /kənˈdens/ *v.*
压缩，精简（内容）

那么，两年前——或者说两个春季学期之前——我教这门课的时候，我总结出了大概五样东西，我认为它们是非常重要的思维工具。这就是——我不得不把内容压缩一点，塞进我那场著名的「思维工具」讲座里。嗯，这里的想法是，《哥德尔、埃舍尔、巴赫》提供了大量的概念工具，用来思考这个复杂的问题：你如何从非自我走到自我？嗯，我先快速地把它们列一下。呃，我们会有

7:17
Isomorphisms, and I'll explain all these terms as we go on. Recursion. I'm going to leave this one mainly up to Curran on the second lecture. paradox.

同构，这些术语我都会在后面一一解释。递归。这一个我主要留给 Curran 在第二讲里讲。嗯，悖论。

7:39
And this is infinity, which and all concepts are very closely linked. And finally, the main subject of for today's lecture is going to be formal systems.

还有这个，无穷——所有这些概念都紧密相连。最后，今天这一讲的主要内容将是形式系统。

04 同构：宽松与严格之分

8:02
All righty. So, first let me go through kind of definitions of these terms. An isomorphism I want you to all be very careful with this because when you start talking to mathematicians, you know, grown-up professional mathematicians, they're going to use the term isomorphism to mean something very, very specific. The way it's used in Gödel, Escher, Bach, the way it's going to be used in this class is very loose. We're going to make very kind of intuitive statements like you know, what's the isomorphism between a car I'm not a great artist here.

注意术语的语域差异：数学中isomorphism指保持运算结构的双射；本课沿用GEB的宽松用法，仅指「结构上的对应」。

好的。那么，首先让我过一遍这些术语的定义。嗯，同构——我希望大家对这个词非常小心，因为当你开始跟数学家交流的时候，你知道，跟那些成熟的职业数学家，嗯，他们用「同构」这个词是指某种非常非常具体的东西。而在《哥德尔、埃舍尔、巴赫》里的用法，也是我们这门课里的用法，是非常宽松的。我们会做一些相当直觉化的表述，比如说，嗯，你知道，一辆车和……的同构是什么——我画画不太行。

8:55

What's the isomorphism between a skateboard and a car? and you know, you might say lots of things like it carries a person, it's it has four wheels. So, what we do is we construct a map, which also has an inverse. And that's the way you think of an isomorphism. You can go either way and preserve information, preserve kind of structure. if you if you really feel like following along, I've in– cluded actually a quote from Douglas Hofstadter. And on a page seven of your of your lecture notes.

滑板和汽车之间的同构是什么？嗯，你知道，你可能会说很多东西，比如它能载人，呃，它——它有四个轮子。所以我们做的事情，就是构造一个映射，而且这个映射还有一个逆映射。这就是——这就是你该怎么理解同构。你可以从任一方向走过去，嗯，并且保留信息，保留某种结构。呃，如果你——如果你真的想跟着看，我其实收录了一段侯世达的原话，嗯，在你们讲义的第七页上。

同构的直觉定义：一个可逆的映射，两个方向都保留结构信息。滑板与汽车的类比说明「对应部分在各自结构中扮演相似角色」。

inverse /ˌɪnˈvɜːrs/ *n.*
逆，逆映射

9:35

He says, and this is in the middle of the page, the word isomorphism applies when two complex structures can be mapped onto each other in a ways that to each part of one structure there's a corresponding part in the other structure. Where corresponding means that the two parts play similar roles in the respective structures. This is how we're going to always use the term isomorphism in this class. If you're taking the abstract algebra class, it's going to mean something a lot more specific and you're going to have a lot more details.

嗯，他说——就在那一页的中间——「同构」这个词适用于这种情况：两个复杂结构之间可以互相映射，使得一个结构的每一部分在另一个结构中都有一个对应的部分。而「对应」的意思是，这两个部分在各自的结构中扮演相似的相似的角色。嗯，这就是我们这门课里始终会使用「同构」这个词的方式。如果你在上抽象代数那门课，它的含义会具体得多，你会接触到多得多的细节。

10:11

You might actually think of these as kind of a what I'll say, but don't worry about worry about it, is a homomorphism. And the idea with a homomorphism is that there are a lot more details here than there are here. and you're For example, there's no steering well. There's a steering well in a car, but there's no steering well specifically in a skateboard. So, if you were to go if you were to create a map from the car to the skateboard, that detail would have to go somewhere else. But, don't worry about that those necessities. But, when I say the term isomorphism, think of equals. And then I'll often use that symbol right there.

嗯，你其实可以把这些看成是一种——我这么说，但你先别太在意——同态（homomorphism）。同态的想法是，这一边的细节要比那一边多得多。嗯，比如说，滑板上并没有方向盘。汽车里有方向盘，但滑板上并没有专门的方向盘。所以，如果你要从汽车映射到滑板的话，这个细节就得放到别的地方去。嗯，不过先别管这些必要条件。当我说“同构”这个词时，你就把它想成“等于”。然后我经常会用右边这个符号来表示它。

同态（homomorphism）是放宽的同构：允许丢失细节的多对一映射。汽车的方向盘在滑板上没有对应物，故二者只能是同态。

homomorphism /ˌhoʊməˈmɔːrfɪzəm/ *n.*
同态（允许丢失细节的结构映射）

05 递归、斐波那契与分形维数

10:55

So, this is going to be really important because it's going to be how we're going to get meaning out of things. And you'll see it a lot coming up over the book. But, first I want to hop on and talk about recursion. Recursion is basically it's seen everywhere but it's kind of a list of instructions which you follow, but then repeat until you've reached kind of a final case. So, suppose you were you're cooking and you had a you have a recursive algorithm for stirring eggs and that would be whirl and then whirl again and keep whirling until essentially everything looks mixed up. That's a very loose way of understanding it, but another way which you all are probably familiar with a much more rigorous in term of mathematics is the Fibonacci sequence.

所以这一点非常重要，因为我们正是靠它来从事物中提取意义。而且，接下来在整本书里你会经常看到它。不过，我想先切换一下，来谈谈递归。递归其实到处都能看到，嗯，它基本上就是一串你要照着执行的指令，然后不断重复，直到到达某种最终情形为止。比如说，假设你在做饭，你有一个搅蛋的递归算法，那就是搅一圈，然后再搅一圈，一直搅到基本上所有东西看起来都混合均匀为止。这是一种很宽松的理解方式，但还有另一种你们大概都很熟悉、在数学上严谨得多的例子，就是斐波那契数列。

递归的两个要素：以自身定义自身＋有终止条件（基例）。搅蛋是操作性递归，斐波那契是定义性递归。

recursion /rɪˈkɜːrʒən/ *n.*
递归（以自身定义自身的过程）

whirl /wɜːrl/ *v.*
旋转，搅动

11:50

This is where you start with two numbers one and one and then you construct the next number by summing the previous two. So, you have that and you have three and you have five and you have eight and so on. And you can create what's called a recursive definition where you define the nth Fibonacci number.

它是从两个数字 1 和 1 开始，然后把前两个数相加来构造下一个数。于是你就得到了这个，然后是 3，然后是 5，然后是 8，依此类推。你可以创建所谓的递归定义，也就是定义第 n 个斐波那契数。

斐波那契数列（1,1,2,3,5,8...）由比萨的列奥纳多于1202年《计算之书》引入西方，广泛出现在植物叶序等自然现象中。

12:20

This is for n greater than or equal to two. And here you define the thing in terms of itself. And this is a classic example of recursion. What it is really itself on a smaller level. I think one of the most exciting applications of recursion are fractals because the way we create fractals is through recursion. So, I don't know if you all have seen this, but the Sierpinski triangle or the Sierpinski gasket is kind of a classic fractal. Here you divide a triangle up into three and then you just repeat the process for infinitely number of an infinite number of times on each remaining triangle.

这里 n 要大于或等于 2。在这里，你用这个东西自身来定义它。这是递归的一个经典例子。它本质上就是它自己在更小规模上的样子。我觉得递归最激动人心的应用之一就是分形，因为我们创造分形的方式正是通过递归。不知道你们有没有见过这个，谢尔宾斯基三角形，或者叫谢尔宾斯基垫片，算是一个很经典的分形。你把一个三角形分成三个，然后对剩下的每个三角形无限次地重复这个过程。

谢尔宾斯基三角由波兰数学家谢尔宾斯基1915年提出：不断挖去中心倒三角，是自相似分形的经典例子。

fractals /ˈfræktlz/ *n.*
分形（维数可为分数的自相似图形）

gasket /ˈgæskɪt/ *n.*
垫片（Sierpinski gasket 谢尔宾斯基垫片/三角）

13:06

And you create these very beautiful kind of mosaic forms. But the nice thing about mathematics is that we can be very precise and do things which we can't do in the real world and that's repeat this infinitely and so on. just for a quick digression and I really don't want to spend too much time on it cuz Karen will do more. why is it called a fractal? Does anyone know? I think it's like a fragment of something. Sure. that it was a term coined by Benoit Mandelbrot in 1977, I believe. It actually refers to its number of dimensions. So, this might be a kind of a mind-bending concept for most of you, but we like to think we live in one, two, or three, or four dimensions.

于是你就得到了这些非常漂亮的、有点像马赛克的图形。但数学的妙处就在于，我们可以非常精确，可以做一些现实世界里做不到的事情，比如把这个过程无限重复下去等等。嗯，稍微跑个题，我真的不想在这上面花太多时间，因为 Karen 会讲得更多。嗯，为什么它叫分形（fractal）呢？有人知道吗？我觉得它像是某个东西的碎片。没错。嗯，这个词是本华·曼德博在1977年提出的，我记得是。它其实指的是它的维数。维度。所以这对你们大多数人来说可能是个挺烧脑的概念，但我们习惯认为自己生活在一维、二维、三维或者四维空间里。

fractal一词由数学家曼德博创造（1975年，源自拉丁语fractus「破碎的」），指维数可为分数的自相似图形。

mosaic /moʊˈzeɪɪk/ *n.*
马赛克，镶嵌图案

digression /daɪˈɡreʃn/ *n.*
离题，跑题的插话

coined /kɔɪnd/ *v.*
创造（新词）；coin a term 造一个术语

mind-bending /ˈmaɪndˌbendɪŋ/ *adj.*
烧脑的，令人费解的

13:54

All integers, right? But my claim is that the Sierpinski gasket actually lives in between one and two dimensions. It lives in like 1.63 something dimensions. but I'm going to help you kind of think about that. And if you if you want to hop along to a page nine, kind of got a recipe for helping you think about dimension. You know what? It's weird because only mathematicians would ever worry about rigorously understanding the concept of what a dimension means. So, here's one way to think about it.

嗯，都是整数，对吧？但我要说的是，谢尔宾斯基三角其实处在一维和二维之间。它存在于大约1.63几维的空间里。嗯，不过我会帮大家理解这一点。如果你们想跟着翻到第九页，那里有个方法，可以帮你思考维度这件事。你知道吗，这挺奇怪的，因为只有数学家才会去操心严格地理解维度到底是什么意思。那么，这里有一种思考方式。

分形维数是本段的反直觉核心：维度不必是整数。谢尔宾斯基三角约1.585维，介于线（一维）与面（二维）之间。

14:30

If you take a line and you double it, you have two copies of the line that you started with. This guy's here and there. If you have a square and you double the sides of the square, you have four copies of the original square. Similarly, and I'm not going to try to draw this cuz it will get too complicated way too fast. If you take a cube and you double each of the sides,

如果你取一条线段，把它加倍，你就得到了两份原来的线段。这一份在这儿，那一份在那儿。如果你有一个正方形，把正方形的边长加倍，你就得到了四份原来的正方形。同样地，我就不画了，因为很快就会变得太复杂。如果你取一个立方体，把每条边都加倍，

用「加倍后得到几份拷贝」定义维度：线段2份、正方形4份、立方体8份，即2^D规律。这是「相似维数」的直观版本。

15:08

You get, if you think about it, eight copies of the original cube. So, if you're perceptive enough, you might kind of realize this action of powers going on here. So, here we had after our doubling process two copies. We had two to the one. Here after our doubling process we had two to the two. After our doubling process here, we had two to the three. Eight. So, this is weird because notice that the cube lives in three dimensions. And the square lives in two dimensions. And the line lives in one dimension.

你想想看，就会得到八份原来的立方体。所以，如果你足够敏锐，可能已经察觉到这里出现了幂次的规律。看，这里经过加倍之后我们得到两份。就是2的1次方。这里经过加倍之后我们得到2的2次方。而这里经过加倍之后，我们得到2的3次方。八。这就奇妙了，因为注意，立方体存在于三维空间中。正方形存在于二维空间中。线段存在于一维空间中。

perceptive /pərˈseptɪv/ *adj.*
敏锐的，观察力强的

15:50

So, this might suggest to you well the relationship that two to the D, where D is the dimension of the space you're living in, equals the number of copies you have after the doubling process. So, let's return to our friend the Sierpinski gasket. If we start here and we imagine doubling each of the sides of the Sierpinski gasket here and here, we're very strangely led to the conclusion that whatever dimension the Sierpinski gasket lives in, it obeys this rule. So, take the logarithm and D times Sorry, this is getting crowded.

所以这可能会让你想到这样一个关系：2的D次方，其中D就是你所处空间的维数，等于加倍之后你得到的份数。那么，我们回到我们的老朋友谢尔宾斯基三角。如果我们从这里开始，设想把谢尔宾斯基三角的每条边都加倍，这里、还有这里，我们就会被引向一个非常奇怪的结论：无论谢尔宾斯基三角存在于几维空间中，它遵循这条规则。所以，取对数，然后 D 乘以……抱歉，这里写得有点挤了。

谢尔宾斯基三角边长加倍后只得3份拷贝，故2^D=3，解得D=log3/log2≈1.585，不是整数——「分形」之名由此而来。

logarithm /ˈlɔːɡərɪðəm/ *n.*
对数

06 悖论三类型：从芝诺到罗素

16:49

Take the logarithm on both sides and solve for D, you'll see that the dimension of the Sierpinski gasket is log three over log two, which is approximately 1.585 on to infinity. So, here's an exact example of something which lives somewhere between one and two dimensions. And I think that's a really cool concept. Moving on for other tools for thinking, we have paradoxes. paradoxes come in all sorts of different flavors. I don't know if some of you have heard of the birthday paradox where it's the idea of okay, what's the probability that someone else in the room has your same birthday?

两边取对数，解出 D，你会看到谢尔宾斯基三角垫的维数是 log 3 除以 log 2，大约等于 1.585，后面还有无穷多位。所以，这就是一个精确的例子，某个东西的维数介于一维和二维之间。我觉得这是个非常酷的概念。嗯，接下来讲讲其他的思维工具，我们有悖论。嗯，悖论有各种各样的类型。不知道你们当中有没有人听说过生日悖论，它的想法是，好，房间里有另一个人和你同一天生日的概率是多少?

生日悖论属「真实性悖论」：23人房间中存在同生日两人的概率已超50%，40人超89%。直觉低估是因为人只想「和我同生日」而非「任意两人」。

17:31

Everybody thinks it's really small, but if you actually work out the mathematics, turns out you actually have a good chance. If you're in a room with over 40 people, you have an extremely high chance of finding someone else with your same birthday. so I've actually listed out this is kind courtesy of Wikipedia and Mr. Quine. we have sort of three variants of a Oops. We have three variants of paradoxes. this is a veridical. And these are things which are true, but they seem paradoxical at first.

大家都觉得这个概率非常小，但如果你真的把数学算一遍，你会发现概率其实相当大。如果你在一个超过 40 人的房间里，你有极高的概率找到一个和你同一天生日的人。嗯，所以我其实列了一下，这要感谢维基百科和蒯因先生。嗯，我们大概有三种变体的……哎呀。我们有三种类型的悖论。嗯，这一种是「真实性悖论」(veridical)。这些是真实成立的东西，嗯，但乍一看它们显得很矛盾。

逻辑学家蒯因（W.V.O. Quine，1962）把悖论分三类：真实性（结论真但反直觉）、谬误性（推理有误）、二律背反（真正的逻辑矛盾）。

veridical /vəˈrɪdɪkl/ *adj.*
真实的，与事实相符的（veridical paradox 真实性悖论）

courtesy of *phr.*
承蒙…提供，来自…（表来源致谢）

18:20

There's falsidical. And I'll give an example of each of these. And then kind of the classic, the one which we are going to be interested in and these are real paradoxes, our antinomies. To give an example of another classic paradox and one which is visited in Gödel, Escher, Bach very early on. It's called Zeno's paradox and the idea is if I want to get from here to my laptop, I first need to walk halfway across the distance. And then if I want to walk the remaining distance, I need to walk half of that.

嗯，还有「谬误性悖论」(falsidical)。我会给每一种举个例子。然后是那种经典的、我们真正感兴趣的一类，这些才是真正的悖论，叫做「二律背反」(antinomies)。再举一个经典悖论的例子，这个悖论在《哥德尔、埃舍尔、巴赫》里很早就被提到了。它叫芝诺悖论，它的想法是，如果我想从这里走到我的笔记本电脑那儿，我首先得走完这段距离的一半。然后如果我想走完剩下的距离，我得走它的一半。

芝诺（公元前5世纪，埃利亚学派）的二分悖论：到达终点前必先走完一半，无限细分似乎使运动无法完成。

falsidical /fɔːlˈsɪdɪkl/ *adj.*
谬误性的（结论假、推理藏有错误的悖论）

antinomies /ænˈtɪnəmiz/ *n.*
二律背反，真正的逻辑悖论

19:05

If I want to walk the remaining distance, I need to walk half of that. And then half of that, half of that. And eventually I get stuck in this infinite loop where it seems like I'm not getting to my laptop. A variant of this paradox is the idea that if I even want to move at all, if my atoms want to pass in space, first they have to go halfway. But before I can go halfway, it's got to go halfway of that half, and halfway of that half, and that half of that half. So Zeno, back in Greece, actually used this to prove that motion was impossible, and that any motion we saw in the universe was an illusion.

> 如果我想走完剩下的距离，我又得走它的一半。然后再一半，再一半。最终我陷入了这个无限循环，看起来我永远到不了我的笔记本电脑那儿。这个悖论的一个变体是这样的想法：如果我想要移动哪怕一点点，如果我的原子想要在空间中移动，它们首先得走完一半。但在我走完一半之前，我得先走那一半的一半，再走那一半的一半，还有那一半的一半。所以古希腊的芝诺其实用这个来论证运动是不可能的，我们在宇宙中看到的任何运动都是一种幻觉。

19:43

So, it's weird. Why? and nobody really could answer Zeno for the longest time. But then it took essentially the development of the understanding of limits in calculus to really get an idea of why this wasn't paradoxical, what rigorously did we mean by an infinite number of steps, what how could we actually get to the cross across the room? It seemed paradoxical, but we knew it had to be true. We knew motion had to be possible. I'm sure when you're all were younger, or even now, you've seen all sorts of kind of false ethical paradoxes where somebody will write out a string of if you take 1 + you take 1 – 1 + 1 – 1 dot, and the person convinces you, well look, if you look in groups of this, these are all zeros, so if you just add a bunch of zeros together, it's not necessarily zero.

> 芝诺悖论直到微积分极限理论成熟才被严格化解：无穷多段之和可以是有限值（1/2+1/4+...=1），故属「真实性悖论」。

> **paradoxical** /ˌpærəˈdɑːksɪkl/
> *adj.*
> 自相矛盾的，悖论式的

> 所以这很奇怪。为什么呢？嗯，而且很长一段时间里没人真的能回答芝诺。但后来基本上是靠微积分中极限概念的发展，才真正让人明白这为什么不是悖论：我们说无穷多个步骤，严格来讲到底是什么意思？我们究竟怎么才能走到房间的另一头呢？它看起来像悖论，但我们知道它必定成立。我们知道运动一定是可能的。嗯，我相信你们小时候，甚至现在，都见过各种各样虚假的伦理悖论，有人会写出一串式子，比如 1 + ……你取 1 − 1 + 1 − 1，等等等等，然后那个人说服你：你看，如果你这样分组来看，嗯，这些全都是零，那么把一堆零加在一起，结果不一定是零。

20:41

But I mean this is an infinite string, right? And we can repeat the pattern. what happens if we add a one? Right? So suddenly we get these weird conclusions where 0 = 1, and they're usually built on kind of doing something illegal involving infinities. And infinity is going to be a very important concept that we'll encounter again and again. Finally, the antinomy. These are the important paradoxes to think about. I once went out to dinner with a bunch of mathematicians. I don't know how I ended up in that, but let me tell you it was kind of frightening. and there was this Korean mathematician who said well you know what? Like most of these questions don't even matter. We don't We don't understand some of the most fundamental things. And the thing he was most interested in and I think what bothers mathematicians the most is the antinomy of the of the liar and Russell's paradox.

> 但我的意思是，这是个无穷的串，对吧？而且我们可以重复这个模式。嗯，如果我们加上一个 1 会怎么样？对吧？于是突然间我们得到这些奇怪的结论，比如 0 = 1，而它们通常都建立在对无穷做了某种非法操作之上。而无穷将会是一个非常重要的概念，我们会一次又一次地遇到它。最后，是二律背反（antinomy）。这些才是真正值得思考的重要悖论。我有一次和一群数学家一起去吃晚饭。我不知道自己怎么会跑到那儿去的，但我告诉你，那还挺吓人的。嗯，当时有位韩国数学家说，你知道吗？其实这些问题大多数根本不重要。我们连一些最基础的东西都还没搞懂。而他最感兴趣的东西，我觉得也是最困扰数学家的东西，就是说谎者的二律背反和罗素悖论。

> 1—1+1—1…是Grandi级数，按不同分组可「得」0或1，谬误在于对发散级数使用结合律——「谬误性悖论」的典型。

21:40

So, the liar's paradox you probably have heard before. And it starts It's based on actually a biblical reference, but it essentially says this sentence is not true. So is it true or is it not true? Well, if it's true then it says of itself that it's not true. So, it's true implies not true. Contradiction. So, if it's not true, then we know that we believe in the law of the excluded middle, which means that things have to either be true or not true that its negation is true. So, if it's not true, then the sentence is true. So, not true implies true.

嗯，说谎者悖论你们大概都听说过。它的起源其实是出自《圣经》里的一句话，但它本质上就是说：这句话不是真的。那么它是真的，还是不是真的呢？如果它是真的，那么它自己说自己不是真的。所以，真推出不真。矛盾。那么如果它不是真的，那我们知道我们相信排中律，也就是说事物必须要么真要么不真，即它的否定为真。所以，如果它不是真的，那这句话就是真的。所以，不真推出真。

说谎者悖论可溯至《圣经·提多书》引用的克里特人「克里特人都说谎」。排中律（非真即假）是推出矛盾的关键前提。

negation /nɪˈgeɪʃn/ *n.*
否定（逻辑术语）

law of the excluded middle *phr.*
排中律（命题非真即假，无第三种可能）

22:36

So we're stuck. The liar paradox still hounds us today. Unlike Zeno's paradox it hasn't been solved. We still don't know how to deal with it. And when we talk about Gödel's theorem the way he proves his result is actually going to be intimately linked with a on this. So, instead of saying I'm not true, it's going to say I'm not provable. And that's going to be a very interesting idea, and we'll explore that a little bit later. The other antinomy I want to look at is Russell's paradox, also known as the barber's paradox, and that's how I'm going to tell it.

所以我们卡住了。说谎者悖论至今仍纠缠着我们。跟芝诺悖论不一样，它还没有被解决。我们仍然不知道该怎么处理它。而当我们谈到哥德尔定理时，他证明这个结果的方式，其实跟这个是密切相关的。跟这个密切相关。所以，它不会说"我不是真的"，而是会说"我是不可证明的"。这会是一个非常有意思的想法，我们稍后会稍微展开讲一讲。我想看的另一个二律背反是罗素悖论，也叫理发师悖论，我打算就用理发师这个版本来讲。

全书关键伏笔：哥德尔把「这句话不真」改造成「这句话不可证明」，从而绕开悖论、得到不完备性定理——真与可证明由此被区分。

hounds /haʊndz/ *v.*
纠缠，穷追不舍地困扰

intimately /ˈɪntɪmətli/ *adv.*
密切地（intimately linked 紧密关联）

23:21

It's the barber's paradox. I think it's a little more friendly.

就是理发师悖论。我觉得它稍微亲切一点。

23:29

So, you have a town, and there's this male barber, and he abides by the rule that he shaves all people and only people who don't shave themselves. So, what does the barber do when his beard is getting as thick as mine? Does he shave himself, or does he not? Well, let's see. So, by definition, the barber only shaves those people who don't shave themselves. So, if he shaves himself, then he doesn't. And if he doesn't shave himself, then by definition, he must shave himself. A variant of this which is which was coined by both Bertrand Russell, Cambridge mathematician and philosopher, and Zermelo, great German logician, is the idea that you can consider the set, let's call it omega, which contains all sets that aren't members of themselves.

> 假设有一个小镇，镇上有一位男理发师，他遵守这样一条规则：他给所有不给自己刮胡子的人刮胡子，而且只给这些人刮。不给自己刮胡子的人。那么，当理发师的胡子长得跟我这么浓密时，他该怎么办呢？他给自己刮胡子，还是不刮？我们来看看。按定义，理发师只给那些不给自己刮胡子的人刮胡子。所以，如果他给自己刮胡子，那他就不该给自己刮。而如果他不给自己刮胡子，那么按定义，他就必须给自己刮。这个悖论有一个变体，是由剑桥数学家和哲学家伯特兰·罗素，以及伟大的德国逻辑学家策梅洛分别提出的，它的想法是：你可以考虑这样一个集合，我们把它叫做omega，它包含所有不是自身成员的集合。

> 罗素1901年发现该悖论并致信弗雷格，动摇了朴素集合论；策梅洛更早独立发现。理发师版本是其通俗化表述。

> **abides by** *phr. v.*
> 遵守，恪守（规则）

24:52

So, remember a set is just a collection of objects. And the mathematicians really believed that set theory was going to be what gave mathematics its ultimate sure and logical foundation. So, let's give an example of a set which contains itself. So, let's think of the set of all things which aren't Joan of Arc. Well, sets aren't people. I mean, they're people, not sets. so, that set of all things which aren't Joan of Arc includes itself because a set can never be a person. So, that set is contained in itself.

> 记住，集合就是一堆对象的汇集。而数学家们真的相信，集合论将会为数学提供最终的、可靠的逻辑基础。逻辑基础。那我们来举一个包含自身的集合的例子。我们来想想"所有不是圣女贞德的东西"构成的集合。嗯，集合不是人。我是说，人是人，不是集合。所以，那个由所有不是圣女贞德的东西构成的集合包含它自身，因为集合永远不可能是一个人。所以那个集合包含在它自身之中。

> 19世纪末逻辑主义纲领（弗雷格、罗素）试图把全部数学奠基于集合论与逻辑，罗素悖论直接引发「第三次数学危机」。

25:29

So, we have a bunch of things in here which are sets which aren't members of themselves. And then we ask the question, is omega an element of itself? And this means is in well, if omega contains itself, but omega by definition only contains things which don't contain themselves, so it can't contain itself. Well, if it can't contain itself, it doesn't contain itself, and that means it should contain itself. Contradiction. this really, really bothered a lot of mathematicians for a long time.

所以，我们这里面有一堆东西，它们是不属于自身成员的集合。然后我们提出这个问题：欧米伽是它自己的元素吗？也就是说，欧米伽是否属于它自己。嗯，那么，如果欧米伽包含它自己，但欧米伽按定义只包含那些不包含自身的东西，所以它不可能包含自己。那么，如果它不能包含自己，它就不包含自己，而这又意味着它应该包含自己。矛盾。嗯，这个问题在很长一段时间里真的、真的困扰着很多数学家。

omega是否属于自身，两个方向都导致矛盾，与理发师悖论结构完全同构——这正是「同构」工具的一次现场示范。

07 无穷的层级与形式系统

26:16

And it's an exact variant on the barber's paradox. So, this is a kind of an interesting thing is to play around with. Finally is the concept of infinity. I can't really talk too much about it. We're going to look at it more, but I want to introduce you guys to the idea that there are multiple types of infinity. So, you have the integers, and you also have the real numbers. And it is true that you cannot create a direct link. You can't match every real number, like 0.333333 Well, 0.35 something random. Pi. Let's just pick pi. You can't put pi directly in connection with a natural number, an integer.

嗯，而且它其实就是理发师悖论的一个精确变体。所以，这是个挺有意思的东西，可以拿来琢磨琢磨。最后是无穷的概念。我没法讲得太深入。我们之后会更多地讨论它，但我想先给大家介绍一个想法：无穷有多种类型。比如你有整数，你也有实数。而事实是，你没法建立一种直接的对应。你没办法把每一个实数都匹配上，比如 0.333333，嗯，0.35 之类随便举的数。圆周率 π。我们就拿 π 来说吧。你没法把 π 直接和某个自然数、某个整数对应起来。

康托尔证明实数无法与自然数一一对应：无穷有大小之分。可数无穷（整数）与不可数无穷（实数）是最基本的两级。

variant /ˈveriənt/ *n.*
变体，变种

27:00

And this is kind of famous Cantor's diagonalization argument. So, somehow there are different degrees of infinity, and the real numbers is a higher degree of infinity. So, that's an important thing to think about. Now we're going to jump ahead to our last tool for thinking, and this is going to be the reason why we ignore the first three chapters of Gödel, Escher, Bach. And it's the idea of a formal system. Problem is these formal systems are boring. and Douglas Hofstadter takes his sweet, sweet time in introducing you to the concept of a of a formal system.

嗯，这就是著名的康托尔对角线论证。所以，无穷不知怎么就有了不同的等级，而实数是更高等级的无穷。所以，这是一件值得思考的重要事情。现在我们要往前跳到最后一个思维工具，这也正是我们跳过《哥德尔、埃舍尔、巴赫》前三章的原因。这就是形式系统的概念。问题在于这些形式系统很无聊。嗯，而且道格拉斯·侯世达在向你介绍形式系统这个概念时不慌不忙、慢条斯理。

对角线论证（康托尔，1891）：假设已列出全部实数，构造一个在第n位与第n个数不同的新数即得矛盾。哥德尔的证明也用了对角线技巧。

diagonalization /daɪˌægənələˈzeɪʃn/ *n.*
对角线法（康托尔证明实数不可数的方法）

takes his sweet, sweet time *phr.*
慢条斯理，不紧不慢地做（含轻微不耐烦意味）

08 MU谜题：符号游戏规则

27:47

So I'm going to try to speed things up because I know you all are smarter than that, and you can get through these concepts very quickly. we're going to play a game. It's called the MU puzzle, M U. and the way you play it is you start with you have a bag of three letters. And you're going to have a rule. You're going to start with you pull two letters out and you get MI. And we have we're going to have four rules, and these are completely strict typographical rules for thinking about for deriving new things that we can pull from our bag.

嗯，所以我打算加快点节奏，因为我知道你们都比那聪明，你们可以很快地掌握这些概念。嗯，我们来玩个游戏。它叫 MU 谜题，M–U。嗯，玩法是这样的：一开始你有一个装着三个字母的袋子。然后你会有一条规则。你一开始从袋子里抽出两个字母，得到 MI。我们会有四条规则，这些都是完全严格的排版规则，用来思考呃，用来推导出我们能从袋子里取出的新东西。

MU谜题是GEB第一章的核心装置：MIU系统只有三个字母、一条公理MI、四条规则，用来让读者体验「在系统内思考」。

typographical /ˌtaɪpəˈɡræfɪkl/ *adj.*
排版上的；此处指只涉及符号形状、不涉含义的

28:37

Our first rule is that if we have an I, so suppose we have MI or we could have anything and then an I, we can tack a U on. So, IU. So, right away we know that we can create M I U. Our second rule is suppose we have M and then a string of letters that are I's and U's since they're in our bag of alphabet or alphabet here, then you're going to get for free MXX. So, just as an example, suppose somehow you had M I, which we do. you're going to get MII for free. Third rule. Suppose you have somewhere along the way you end up with a cluster of three I's.

> **tack** /tæk/ *v.*
> 附加（tack on 在末尾加上）

嗯，我们的第一条规则是：如果我们有一个 I，比如说我们有 MI，或者前面是任何东西，然后结尾是一个 I，我们就可以在后面加上一个 U。所以就变成 IU。所以我们马上就知道，我们可以造出 M I U。我们的第二条规则是：假设我们有 M，后面跟着一串由 I 和 U 组成的字母，因为它们就在我们的字母袋，或者说这里的字母表里，那你就可以免费得到 MXX。那么举个例子，假设你手上有 M I，我们确实有。嗯，那你就免费得到了 MII。第三条规则。假设你在某个步骤中得到了连在一起的三个 I。

29:51

They don't have to be at the end, they can be anywhere. Just needs to be three I's all together. And you can replace all three of those I's, they're equal to a U. So,

它们不一定要在末尾，可以在任何位置。只要是三个 I 连在一起就行。然后你可以把这三个 I 全部替换掉，它们等价于一个 U。那么，

30:10

And our final rule is that if we have a double pair of U's, we can drop them and they just go away. So, somehow if we had M U, we could just have M. Now, you have these rules, you have these letters, you start with one guy. He's going to be our axiom. An axiom is a starting point for reasoning, for applying these rules. The game is, can you get MU? Starting from MI and then using only these four rules, can you get MU? I will give $20 to the first person who can derive MU that's in this room. Only applying these four rules and starting directly from MI.

我们最后一条规则是：如果我们有连续两个 U，就可以把它们去掉，它们直接消失。所以，如果我们不知怎么得到了 M U U，那我们就可以直接得到 M。现在，你有了这些规则，有了这些字母，你从一个起点开始。它将是我们的公理。公理就是推理的起点，是应用这些规则的出发点。游戏就是：你能得到 MU 吗？从 MI 开始，只使用这四条规则，你能得到 MU 吗？这个房间里第一个能推导出 MU 的人，我给他 20 美元。只能应用这四条规则，而且必须直接从 MI 出发。

公理＝推理的起点。20美元悬赏其实很安全：MU不可导出——I的个数模3是不变量（初始为1，规则只能翻倍或减3），永远不可能变为0。

axiom /ˈæksiəm/ *n.*
公理，推理的起点

derive /dɪˈraɪv/ *v.*
推导出，导出

31:14

Just to give you an idea of where you might be going, where you might be playing, just going off of our rules, we already saw that if we had MI, we can get MIU. We also saw that using rule two, this is using rule one, we can get M II. We saw if we have anything like that, we can repeat it twice, so we can get MIUIU. That's applying rule two again. and so on. Leave this Leave this as a puzzle. Take your time with it. You'll be working on it for a few hours. But first person that's in this room derive MU from this gets $20.

为了让你们大致了解可以往哪个方向走、可以怎么玩，嗯，就按我们的规则来说，我们已经看到，如果有 MI，就能得到 MIU。我们还看到，用规则二——这里用的是规则一——我们可以得到 MII。我们看到，如果有类似这样的东西，就可以把它重复两遍，于是能得到 MIUIU。这是再一次应用规则二。嗯，以此类推。就把这个当作一道谜题吧。慢慢来。你可能要花上几个小时来琢磨它。但这个房间里第一个从这里推导出 MU 的人，能拿到 20 美元。

四条规则小结：末尾I后加U；M后的串整体翻倍；三个连续I换成U；两个连续U删去。可推出MIU、MII、MIUIU，但推不出MU。

32:03

Yes. Fourth rule only applies to you? Yes, fourth rule only applies to two Us. So, yes, if you have two Us, you can remove them. You can subtract them. All right. And once again, I do urge everyone to buy the book. these rules are listed explicitly in the chapter. and you might get gain some insight on how to derive what you want here.

是的。第四条规则只适用于 U 吗？是的，第四条规则只适用于两个 U。所以，是的，如果你有两个 U，你就可以把它们去掉。你可以把它们减掉。好的。再说一次，我确实强烈建议大家去买这本书。嗯，这些规则在那一章里都有明确列出。嗯，而且你可能会从中获得一些启发，明白该怎么推导出你想要的结果。

32:38

So, why is this interesting? I mean, it's a it's we're just playing with letters and strings and things like that. Well, although this seems pretty meaningless and kind of dumb, does anybody feel like when they're just looking at this game and looking at these rules that they're just essentially playing around with algebra that they learned, you know, in middle school or high school? That really what we're doing here is we've got some statements like 2 + 2 = 4, and we all learned that we have a typographical rule, for when we have an equal sign like that, we can add one to both sides and preserve equality. So, something we have 2 + 3 = 5.

关键转折：MIU游戏与中学代数「等式两边同加1」在结构上是同一回事——数学操作本身就是排版规则的游戏。

那么，这为什么有意思呢？我是说，我们不就是在玩字母和字符串之类的东西吗。嗯，虽然这看起来相当没有意义，甚至有点傻，嗯，但有没有人觉得，当你看着这个游戏、看着这些规则的时候，你本质上就是在玩你在初中或者高中学过的代数？我们在这里做的事情，其实就是有一些陈述，比如 2 + 2 = 4，而我们都学过一条排版规则，嗯，就是当我们有这样一个等号时，我们可以在两边同时加上 1，并且保持等式成立。于是我们就得到了 2 + 3 = 5。

33:33

So, really what mathematics reduces to is just playing around with systems of this form and applying these rigorous kind of typographical rules. Except here there's doesn't seem to be any meaning. It's just meaningless. One of the important questions we're going to address in this class is how do things gain meaning? How do we go from meaningless to meaning? this obviously seems to have meaning, but I want you to ask yourself why. kind of before we proceed, it's necessary, it's my duty, to do the boring task of writing down I just a few definitions of things which you can call these. So, you have words. So, we already saw axiom.

所以，数学归根结底其实就是在摆弄这种形式的系统，并且应用这些严格的排版式规则。只不过在这里似乎没有任何意义。它就是无意义的。我们这门课要探讨的一个重要问题就是，事物是如何获得意义的？我们如何从无意义走向有意义？嗯，这个（等式）显然看起来是有意义的，但我希望你们问问自己，为什么。嗯，在我们继续之前，有必要——这是我的职责——做一件无聊的事，就是写下，嗯，一些关于这些东西的定义，也就是你们可以怎么称呼它们。所以，有一些术语。我们刚才已经见过公理了。

「意义从哪里来」是本课中心问题：形式系统内部只有符号，意义来自我们在符号与世界之间建立的同构映射。

reduces to *phr. v.*
归结为，简化为

34:18

That's definition. You call any of these guys a string. So, so a string is just any ordered sequence of in this case M's and U's.

那就是定义。这些东西里的任何一个你都可以叫做「字符串」。所以，字符串就是任意一个有序的序列，在这个例子里就是由 M 和 U 组成的。

34:48

We already met an axiom. An axiom is a starting point. It's your first thing that you can apply the rules to. So, and this has actually has a lot to do with mathematical logic because in math logic, the idea is that we start from really primitive things which seem obvious like the successor of zero is one. and then we work from that concept and we derive all these truths of number theory and mathematics. here your axiom is MI and you're trying to prove the theorem and that's kind of our next guy here. we're trying to prove the theorem of MU.

我们已经认识了公理。公理是一个起点。它是你能够对其应用规则的第一个东西。所以，这其实和数理逻辑有很大关系，因为在数理逻辑里，思路是我们从真正原始的、看起来显而易见的东西出发，比如零的后继是一。嗯，然后我们从这个概念出发，推导出数论和数学中的所有这些真理。嗯，在这里你的公理是 MI你想要证明这个定理，这就有点像我们接下来要讲的下一个点。嗯，我们试着去证明 MU 这个定理。

「零的后继是一」出自皮亚诺公理（1889）。现代数理逻辑正是从这类极简公理出发，重建数论与数学的全部真理。

successor /səkˈsesər/ *n.*
后继，后继数（皮亚诺算术术语）

35:38

So, a theorem is basically a string which results at the end of a derivation.

所谓定理，基本上就是一串在推导结束时得到的字符串。

derivation /ˌderəˈveɪʃn/ *n.*
推导，推导过程（形式化证明）

36:00

And a derivation is like a proof. For those of you who have done geometry, when you're saying, "Okay, well, this triangle is congruent to this triangle because of side angle side and things like that." Those are you're deriving you're making rigorous justifications for your leaps in logic. So, here our rigorous justification that MIU was a theorem was that, well, we applied typographical rule number one. That's a rigorous leap in logic and we got to this theorem. And you can just call these four rules here these are rules of inference.

而推导就像是证明。对于学过几何的同学来说，当你说"好，这个三角形和那个三角形全等，因为边角边"之类的，就是这种情况。你是在推导，是在为你逻辑上的跳跃给出严格的论证。所以在这里，我们证明 MIU 是定理的严格论证就是：我们应用了第一条排版规则。那是一次严格的逻辑跳跃，于是我们得到了这条定理。你可以把这四条规则称作推理规则。

推导（derivation）＝形式化的证明：每一步都由推理规则严格辩护，正如几何证明中引用「边角边」定理。

congruent /ˈkɑːŋɡruənt/ *adj.*
全等的（几何术语）

rules of inference *phr.*
推理规则

36:45

And logic and a lot of things that you'll play around with, you know, eventually on SATs and things like that, are, you know, if you have if you have the statement that P implies a statement Q. So, if it's cloudy, then it will rain. You have you have that this is kind of equivalent to I should use a different arrow here. To not Q implies not P. and these are really nice because they're just typographical rules. When you see something, like when you have well, I've got M followed by any string of letters, well, then I can double it. That's a rule of inference. Just like this is a rule of inference. If I have P implies Q I can always replace that.

P→Q等价于¬Q→¬P（逆否命题），是命题逻辑最常用的推理规则之一。讲者借此说明：逻辑推理规则本质上也是「排版规则」。

逻辑，以及很多你以后会接触到的东西，比如说，将来考 SAT 之类的时候会遇到的，就是，如果你有这样一个陈述：P，P 蕴含陈述 Q。比如说，如果天是阴的，那么就会下雨。你会发现，这其实等价于——我这里该用另一种箭头。等价于：非 Q 蕴含非 P。嗯，这些规则非常好，因为它们纯粹是排版规则。当你看到某个东西，比如说，我有一个 M 后面跟着任意一串字母，那我就可以把它翻倍。这就是一条推理规则。就像这条也是推理规则一样：如果我有 P蕴含 Q，我总是可以把它替换掉。

09 跳出系统与元思考

37:35

It's completely equivalent to not Q's implies not P. so But for those of you who are scrambling away because you want $20 really fast, I want you to take a break cuz once again, should focus on what we're what we're saying right now. And we're going to talk a little bit about jumping outside the system. And this is kind of the cool renegade stuff that Hofstadter fills his book with. And it's the idea that as you're playing around with this, you're right now you're just playing a game.

「跳出系统」是GEB的标志性概念：智能的特征之一是能停下机械操作、审视系统本身，而不是无止境地在规则内打转。

renegade /ˈrenɪɡeɪd/ *adj.*
叛逆的，离经叛道的

scrambling /ˈskræmblɪŋ/ *v.*
手忙脚乱地抢做，争抢

它完全等价于"非 Q 蕴含非 P"。嗯，不过对于那些急着想赶快赚到 20 美元的同学，我希望你们先歇一歇，因为你还是应该专注在我们现在讲的内容上。嗯，接下来我们要稍微聊聊跳出系统。这就是霍夫施塔特在他书里塞满的那种很酷的离经叛道的东西。嗯，接下来我们要稍微聊聊跳出系统。这就是霍夫施塔特在他书里塞满的那种很酷的离经叛道的东西。这个想法是说，当你在摆弄这些东西的时候，你现在只是在玩一个游戏。

38:05

And what mathematicians and what anybody human does is when they feel like they're caught in loops, just cranking through pages of algebra and they're not getting anywhere humans are intelligent enough to stop. They exit the system and they say I don't know. I don't think this is going to go anywhere or well, let me think about why I'm not getting or I how might I get MU? You know, maybe it has something to do with numbers of I's and U's or things like that. I'm not sure. You start doing what I like to call meta thinking.

而数学家、以及任何一个人所做的事情是：当他们感觉自己陷入了循环，一页一页地机械推演代数却毫无进展时，人是足够聪明的，会停下来。他们退出这个系统，然后说：我不知道，我觉得这样下去不会有结果；或者说，让我想想为什么我得不到，或者我要怎样才能得到 MU？你知道，也许这跟 I 和 U 的数量有关之类的。我也不确定。你开始做我喜欢称之为"元思考"的事情。

元思考（meta-thinking）：不在系统内应用规则，而是对系统整体做判断，如「也许MU与I的个数有关」——这正是解开谜题的钥匙。

cranking through *phr. v.*
机械地大量做完（乏味的工作）

38:45

You're not thinking in the system applying typographical rules, applying rules of inference to existing strings axioms and getting theorems. That's thinking inside the system. That's just thinking. Meta thinking involves you leaping outside the system and making judgments about it. Thoughts which cannot be expressed as any just normal typographical rule within the system. You're doing meta thinking. What my one of my favorite parts of this of this section of in Gödel, Escher, Bach is when Hofstadter says, and if once again, stop driving to drive MU, try to turn to page 24 in your lecture notes.

你不再是在系统内部思考——不再是应用排版规则、把推理规则用在已有的字符串、公理上来得到定理。那才是在系统内部思考。那只是普通的思考。元思考则是让你跳到系统之外，对这个系统本身作出判断。这些想法无法用系统内部任何普通的排版规则表达出来。这时你就是在做元思考。我最喜欢的部分之一，就是《哥德尔、埃舍尔、巴赫》这一节里，侯世达说的那段话——如果你又一次陷在推导 MU 的死循环里，呃，请翻到讲义第 24 页。

39:34

Oops. It's in my syllabus. Let me get that. No worries. Page 24. Hofstadter kind of uses this as like a as a life lesson. He says, "Look, of course there are cases when only a rare individual will have the vision to perceive a system which governs many people's lives. A system which had never before even been recognized as a system. And such people often devote their lives to convincing other people that the system really is there and that it ought to be exited from." So, that's if our social customs and our kind of cultures are really just formal games. You know, we say hello, we shake your hand. That's an instance of a formal rule which we all follow.

哎呀。它在我的教学大纲里。我找一下，别急。第 24 页。侯世达把这一点当成一种人生教训。他说："你看，当然，有些情况下只有极少数人才有那种眼光，能看出一个支配着许多人生活的系统，一个此前从未被人认作"系统"的系统。这样的人往往用一生去说服别人：这个系统确实存在，而且我们应该从中跳出来。"所以说，如果我们的社会习俗和文化其实只是一些形式游戏——比如我们打招呼、握手，这就是我们都在遵循的一条形式规则的实例。

侯世达把形式系统扩展为社会隐喻：习俗（如握手）是人人遵守的形式规则，只有少数人能「看见」系统并选择退出。

syllabus /ˈsɪləbəs/ *n.*
教学大纲，课程提纲

perceive /pərˈsiːv/ *v.*
察觉，看出

40:21

But, you know, every once in a while you get somebody who says, "Ah, I don't want to shake your hand. I'm going to exit the handshaking formal system." But, of course, there are much more radical examples of this, like I said Karl Marx and communism, you know, he viewed this idea of like "Well, look, you've got these people who are collecting money and property and you know, they're getting someone else to do all the work and they're oppressing this whole class of people. Can't people recognize the system?" So, then people like Karl Marx and Fred Engel like start writing in pamphlets encouraging people to overthrow governments, etc. Because they viewed a system and they said, "Look, we need to exit the thinking system. We're intelligent beings, we can think on a higher level." of course, I'm not trying to promote communism here. Just showing you an example of historical interest.

但你知道，时不时就会有人说："啊，我不想跟你握手。我要退出握手这个形式系统。"呃，当然，还有更激进的例子，比如我说过的卡尔·马克思和共产主义。他的看法大致是："你看，有这么一群人在敛聚金钱和财产，而且他们让别人替他们干所有的活，还压迫着这一整个阶级的人。难道人们看不出这是个系统吗？"于是像卡尔·马克思、弗里德里希·恩格斯这样的人开始写小册子，鼓动人们推翻政府等等。因为他们看到了一个系统，然后说："你看，我们得跳出这套思维系统。我们是有智能的存在，我们可以在更高的层面上思考。"呃，当然，我不是在这里宣扬共产主义，只是给你们举一个有历史意义的例子。

马克思与恩格斯1848年发表《共产党宣言》。讲者以此为例：识别出支配众人生活的系统并号召退出，是社会层面的「元思考」。

oppressing /əˈpresɪŋ/ *v.*
压迫

pamphlets /ˈpæmfləts/ *n.*
小册子，政论传单

overthrow /ˌoʊvərˈθroʊ/ *v.*
推翻（政权）

41:12

You know, anarchism, socialism today, working people's, the media. Nowadays, I think it's one of the most popular things to people say for people to say is like, "Well, you know, it's just the media trying to do this." Before we used to never like just refer to this entity as the media. The media is trying to obscure our understanding of this. The media is trying to scare us. Also, you know, the government. The government's responsible. Of course, a classic example is also what Karl Marx said, you know, the church. They're the opiate of the masses, is what he said.

呃，你知道，还有无政府主义、社会主义，劳动人民，还有媒体。如今我觉得人们最爱说的一句话就是："哎，你知道的，这不过是媒体想搞出来的效果。"以前我们从来不会把"媒体"当成这么一个整体来指称。媒体在试图遮蔽我们对这件事的理解。媒体在试图吓唬我们。呃，还有政府。都是政府的责任。呃，当然，还有一个经典的例子是就像卡尔·马克思说的，你知道，教会。他说过，宗教是人民的鸦片。

「宗教是人民的鸦片」出自马克思《〈黑格尔法哲学批判〉导言》（1843），此处被用作「指认系统」话语的经典案例。

obscure /əbˈskjʊr/ *v.*
遮蔽，使模糊（此处作动词）

opiate /ˈoʊpiət/ *n.*
鸦片剂；使人麻木之物（the opiate of the masses 人民的鸦片）

41:46

And also school. School's my favorite example of, you know, a system which people have encouraged you to exit from. It's like, well, you know, it's just a daycare that we have and we don't actually want kids to learn and grow up. and this has inspired a lot of new free–thinking educational movements, like the Montessoris and things like that. and I really want you guys to think about in your daily actions, am I living perhaps in a in a kind of formal system which is acting in a similar way?

> 还有学校。学校是我最喜欢的例子，你知道，一个大家鼓励你去跳出来的系统。就像是，你知道，它其实就是个托儿所，我们并不真的希望孩子学习和长大。嗯，这也激发了很多新的自由思想教育运动，比如蒙特梭利之类的。嗯，我真的希望你们在日常行为中想一想：我是不是正生活在某种以类似方式运作的形式系统里?

蒙台梭利教育法由意大利医生玛丽亚·蒙台梭利创立，强调儿童自主探索，常被视为对传统学校「系统」的一种退出。

42:12

Try to do some meta–thinking, thinking on a higher level. and is it worth being exiting that system? Hofstadter kind of classifies these three levels of thinking. and he likes to call it a mechanical mode when you're doing the normal games of the system, an intelligent mode, and then just an un–mode. Un–mode is when you just kind of reject the system. He calls it the Zen way of approaching things. And this is something we like to talk about a little more. I want to quickly introduce you to another Well, first of all, I want to talk about a concept of what we've previously mentioned is, you know, we're eventually going to be talking about artificial intelligence.

> 试着做一些元思考，在更高的层面上思考。嗯，然后想想：跳出那个系统值不值得？嗯，侯世达把思考分成三个层次。嗯，当你在按系统的常规玩法行事时，��喜欢称之为机械模式（mechanical mode），然后是智能模式（intelligent mode），再然后就是无模式（un–mode）。无模式就是你干脆拒绝这个系统。他称之为禅式的处事方式。这个我们后面还想多聊一点。再多聊一点。我想快速给你们介绍另一个，嗯，好吧，首先我想聊一个我们之前提到过的概念，你知道，我们最终会讲到人工智能。

侯世达区分三种模式：机械模式（系统内操作）、智能模式（元思考）、无模式（Un–mode，禅式地拒绝系统本身的框架）。

43:00

And it's weird because humans really like to say that their thoughts are logical. We like to say that we do think in this manner, but a lot of times we don't. We like to use kind of just inference about just collective events. Like one of our favorite tools of thinking is induction. Well, you know, the sun has risen all these previous days, sure it'll rise tomorrow. and there's no real formal line of logic that's saying that, well, sun rise yesterday and it thus it will rise tomorrow. And I want you to think of whether or not human our thoughts are actually just computations in a formal system much like MIU or P implies Q and things like that.

> 很奇怪的是，人类特别喜欢说自己的思维是合乎逻辑的。我们喜欢说我们就是这样思考的，但很多时候并不是。我们其实喜欢基于一堆事件做某种推断。比如我们最喜欢的思维工具之一就是归纳。你知道，太阳前面这些天都升起来了，那明天肯定也会升起来。嗯，其实并没有真正形式化的逻辑链条说，太阳昨天升起了，所以明天也会升起。我希望你们想一想，人类的思维是不是其实只是某个形式系统里的运算，就像 MIU 系统，或者「P 蕴含 Q」之类的东西。

> 休谟早在18世纪就指出：归纳（「太阳每天都升起，故明天也会」）没有逻辑必然性——人类思维并不像自诩的那样「合逻辑」。

> **induction** /ɪnˈdʌkʃn/ *n.*
> 归纳法（由个例推出一般结论）

10 pq系统与意义的诞生

43:47

And that's going to bring me to another formal system which I have to mention just because in chapter four he's going to refer to it. And it's going to lead us to this kind of interesting line of dialogue of when a formal system with meaningless symbols gains meaning. and it's called the PQ system. We're going to have three new letters, well, three new characters. It's not going to be P Q and hyphen. And you've actually got an infinite number of axioms here. And when you've you've got a definition.

> 嗯，这就把我引向另一个形式系统，我必须提一下，因为第四章里他会引用它。它。呃，这会引出一条很有意思的讨论线索：一个由无意义符号构成的形式系统是何时获得意义的。嗯，它叫做 pq 系统。我们会有三个新字母，好吧，三个新符号。它们是 p、q 和连字符。而且这里你其实有无穷多条公理。然后你有一个，呃，一个定义。

> pq系统是GEB第二章的例子：只有p、q、连字符三种符号，看似无意义，实则每条定理都恰好对应一个正确的加法等式。

44:41

And that's that if you know XP hyphen and I'm going to kind of make sure I have just an underlined P. Q X And this is going to be an axiom.

就是说，如果你有 x p 连字符——我要确保我这里只写一个带下划线的 p。嗯，q x，这就是一条公理。

45:07

Whenever X is just a string of hyphens. So it's just some string of hyphens. So what's this saying? It's saying that well, if you have something like this Well, X here was two hyphens. So we know that's an axiom. All right, it's a little different than in MIU. Seems just as meaningless. and we're going to have different forms for manipulating and playing around with this. and one rule is that if you have X, Y, and Z which are just hyphen strings XP Y Q Z, then you can derive you're given for free the statement XP Y hyphen Q Z hyphen.

hyphens /ˈhaɪfnz/ *n.*
连字符

只要 x 是一串连字符就行。所以它就是某个连字符串。那这在说什么呢？它是说，如果你有像这样的东西——这里的 x 是两个连字符。那我们就知道这是一条公理。好，这跟 MIU 有点不一样。看上去同样毫无意义。嗯，我们还会有不同形式的规则来操作和摆弄它。嗯，其中一条规则是：如果你有 x、y、z，它们都是连字符串，形如 x p y q z，那么你就可以推出——白送给你——这个式子：x p y 连字符 q z 连字符。

46:23

Seems meaningless. But what does it remind you of? we've got this axiom. We in fact have a whole infinite list of axioms. And maybe you've noticed that you've got two hyphens here. One hyphen here. Got three hyphens here. Now, what does this do? Yeah, exactly. I mean, what it what it does is it says that well, if this works, right? So, let's apply this rule here and we'll apply this rule here. So, we can take this and get for free that hyphen P hyphen we can add another hyphen Q Now, we had three hyphens here but this rule says we can tack on another hyphen.

看着毫无意义。但它让你想到什么？嗯，我们有这条公理。事实上我们有一整个无穷的公理表。也许你已经注意到，这里有两个连字符。这里有一个连字符。这里有三个连字符。那么，这条规则做了什么？对，正是。我的意思是，它做的事情是说：如果这个成立，对吧？那我们把这条规则用在这里，再把这条规则用在这里。于是我们可以拿这个，白得到：连字符连字符 p 连字符——我们可以再加一个连字符——q。本来这里是三个连字符，但这条规则说我们可以再接上一个连字符。

47:27

What does that say? this seems to say that 2 + 2 equals 4. So, I want you to realize that the symbolism which mathematicians have been using and what you've grown up learning is just shorthand, meaningless notation. Yeah? Those two backwards one and two could be two and one. Well, yeah, no. What I meant to say here is that we seem to be inferring this rule that hyphen string one plus hyphen string two always equals hyphen string three. and the And so, you just one here refers to a whole string of hyphens.

> 这说的是什么？嗯，这似乎在说 2 + 2 = 4。所以我希望你们意识到，数学家一直在用的、你们从小学到的那套符号体系，不过是一种速记的、本身无意义的记号。嗯？那两个反过来的一和二，也可以是二和一。嗯，是的，不过——我这里想说的是，我们似乎在归纳出这样一条规则：连字符串一加上连字符串二，总是等于连字符串三。嗯，所以这里的「一」指的是一整串连字符。

揭晓：--p--q----同构于2+2=4。阿拉伯数字与加号本身也只是速记符号——数学记号的意义同样来自约定与同构。

shorthand /ˈʃɔːrthænd/ *n.*
速记，简写记法

inferring /ɪnˈfɜːrɪŋ/ *v.*
推断，推知

48:10

And two refers to a string of hyphens like Y here. Or better yet, I could say X + Y = Z here. And What makes this system different than MIU? Does anyone have any ideas? Why do you suddenly care a little more about this system than MIU? Other than the fact that you have 20 bucks going on the line for deriving MIU.

> 「二」指的是像这里的 y 那样的连字符串。或者更好的说法是，我可以写成 x + y = z。那么，这个系统跟 MIU 有什么不同？有人有想法吗？为什么你突然会比对 MIU 更在意这个系统？除了因为推导 MIU 有 20 块钱悬赏这一点之外。

48:37

Anybody? What about this fact that I've just kind of showed you this equivalence here? I've You now, instead of applying these kind of typographical rules, I've showed you that well, you can also take this as 2 + 2 = 4. And then you're going to say, "Aha! Well, now I can do all sorts of things like Now that I've discovered the meaning of the PQ hyphen system, I can go ahead and just create all sorts of new theorems and starting from any of our axioms. And you might even be tempted to say, "Well, I know it's obvious.

有人吗？那我刚给你们展示的这个对应关系呢？现在，你不再只是应用那些排版规则，我给你们展示了：你也可以把它读作 2 + 2 = 4。然后你就会说：「啊哈！现在我可以干各种各样的事了——既然我发现了 pq 连字符系统的意义，我就可以从任何一条公理出发，造出各种各样的新定理。」你甚至可能忍不住说：「这不是明摆着的嘛。

49:22

I know that 2 + 2 + 2 = 6. And I've discovered this isomorphism between Ps and Qs and pluses and equal signs. So, I'm tempted to say that hyphen P hyphen P hyphen Q hyphen. That's a lot of hyphens. What's wrong with this? Does anyone see a problem? Yes. Exactly. Exactly. It doesn't follow the rule. The rules I told you in the axioms which you start from, you only ever have one P and one Q. This is not even what we call So, this is not what we'll refer to as a well–formed formula.

我知道 2 + 2 + 2 = 6。我已经发现了 p、q 跟加号、等号之间的同构。」嗯，所以我很想写出连字符 连字符 p 连字符 连字符 p 连字符 连字符 q 连字符 连字符 连字符 连字符 连字符 连字符。这可真是一堆连字符。这有什么问题？有人看出问题了吗？请说。没错，没错。它不符合规则。我告诉你们的规则和你们出发用的公理里，永远只有一个 p 和一个 q。这甚至都不算我们所说的——所以这不是我们所说的合式公式（well–formed formula）。

合式公式（well–formed formula）：符合语法规则的符号串。虽然「翻译」成2+2+2=6为真，但该串不是系统能生成的——意义不能倒逼系统。

well–formed formula *phr.*
合式公式（符合形式系统语法规则的符号串）

50:34

So, you have to be really careful with what meaning means and when you try to create an isomorphism between what you know about addition and the formal systems you play. Try to come up with a alternative interpretation. We could have just interpreted these P's, Q's, and hyphens as you know, we're going to call P and we're going to say that's horse and Q and that's apple and you know, one hyphen is happy and you know, two hyphens is happy and so on. So, suddenly we have an interpretation for this string. It's not 2 + 2 = 4, but it's happy horse happy apple happy.

> 所以你必须非常小心「意义」到底意味着什么，以及当你试图在你所了解的加法和你所玩的形式系统之间建立同构时。试着想出一种别的解释。我们完全可以把这些 p、q 和连字符解释成，你知道，我们说 p 就是「马」，呃，q 就是「苹果」，然后一个连字符是「开心」，两个连字符是「开心 开心」，以此类推。那么突然之间，我们对这个串就有了一种解释。它不是 2 + 2 = 4，而是「开心 开心 马 开心 开心 苹果开心 开心 开心 开心」。

「快乐马苹果」的解释说明：同一形式系统可有多种解释，意义并非系统固有；只有当解释使全部定理映射为真命题时，同构才有价值。

51:40

Doesn't mean anything. but it's an interpretation and there's no reason not to make that interpretation. Perhaps the horses this is actually more sensible than addition. I mean, first of all, when we do addition, we're representing these numbers in a in base 10 because we have 10 fingers. But horses don't have 10 fingers, and numbers written in base 10 don't mean anything to horses, but perhaps happy horse apple really makes much more sense to a horse. so we're going to kind of throw out, and I have to be a little rushed about this.

> 没什么意义。嗯，但这是一种解释，而且没有理由不这么解释。也许对马来说，这其实比加法更合理。我是说，首先，我们做加法时是用十进制来表示这些数的，因为我们有十根手指。但马没有十根手指，用十进制写出来的数对马来说毫无意义，而也许「开心马 苹果」对一匹马来说才真正说得通。嗯，所以我们要抛开——这部分我得讲快一点。

反直觉一击：十进制只是因为人有十根手指的约定，对马毫无意义——记号的「自然性」纯属习惯，而非事物本质。

11 现实是形式系统吗

52:17

Be thinking about where does meaning come from? How do we actually assign meaning to meaningless symbols? Cuz that's the goal here. We're going to go from meaningless symbols in mathematics to meaning, and then we're going to try to create nice isomorphism between the universe and our formal systems. And this leads me, you know, perfectly into this idea of, you know, is reality a formal system? and if you go to page 29 in your notes, you've got this kind of long quote stretches onto 30. I'll go ahead and start reading. It's at the bottom.

嗯，想一想：意义从哪里来？我们究竟是怎样给无意义的符号赋予意义的？因为这正是我们的目标。我们要从数学中无意义的符号走向意义，然后我们要试着在宇宙和我们的形式系统之间建立漂亮的同构。这就非常自然地把我引向这个想法：现实本身是不是一个形式系统？嗯，如果你们翻到讲义第 29 页，会看到一段挺长的引文，一直延伸到第 30 页。我来读一下吧。就在这页底部。

52:58

It says, "Can all of reality be turned into a formal system? In a very broad sense, the answer might appear to be yes. One could suggest, for instance, that reality is itself nothing but one very complicated formal system. Its symbols do not move around on paper, but rather in a three-dimensional vacuum space. They are the elementary particles of which everything is composed. Tacit assumption that there is an end to the descending chain of matter, that the expression elementary particles makes sense.

它说：「全部现实能被变成一个形式系统吗？在非常宽泛的意义上，答案似乎可以是肯定的。比如说，有人可能会提出，现实本身不过是一个极其复杂的形式系统。它的符号不是在纸上移动，而是在三维的真空空间里移动。这些符号就是构成万物的基本粒子。（这里有一个默认的假设：物质向下细分的链条是有尽头的，也就是说「基本粒子」这个说法是有意义的。）

引文提出宏大类比：现实本身是一个形式系统——符号是基本粒子。「存在最底层粒子」是这一说法未言明的预设（tacit assumption）。

Tacit /ˈtæsɪt/ *adj.*
心照不宣的，未言明的（tacit assumption 隐含假设）

elementary particles *phr.*
基本粒子

53:27

The typographical rules are the laws of physics, which tell how We're on page 29, if you just want to catch up. the typographical rules are the laws of physics, which tell how, given the positions and velocities of all the particles at a given instant, to modify them, resulting in a new set of positions and velocities belonging to the next instant. So, the theorems of this formal grand formal system are the possible configurations of particles at different times in the universe. The sole axiom is, or perhaps was, the original configuration of all the particles at the beginning of time.

排版规则就是物理定律，它告诉你如何——我们在第 29 页，如果你想跟上进度的话。嗯，排版规则就是物理定律，它告诉你，在给定某一瞬间所有粒子的位置和速度后，如何改变它们，从而得到属于下一瞬间的一组新的位置和速度。所以这个宏大形式系统的定理，就是宇宙中不同时刻粒子可能的组态。唯一的公理是——或者说曾经是——时间开端处所有粒子的初始组态。

类比展开：排版规则＝物理定律，定理＝宇宙各时刻的粒子构型，唯一公理＝时间起点的粒子初始排布（大爆炸时刻）。

velocities /vəˈlɑːsətiz/ *n.*
速度（物理术语，含方向）

configurations /kənˌfɪgjəˈreɪʃnz/ *n.*
构型，排布方式

54:01

This is so grandiose a conception, however, that it has only the most theoretical interest. And besides, quantum mechanics and other parts of physics can at least cast at least some doubt on even the theoretical worth of this idea. Basically, we are asking if the universe operates determinist– deterministically, which is an open question. You know, this it's I think it was Laplace who said, "Well, look, if you were to give me the position and momentum of every particle in the universe, I could tell you the rest of the future."

然而这个构想过于宏大，以至于它只有最纯粹的理论意义。而且，量子力学和物理学的其他部分，至少能对这个想法哪怕只是理论上的价值投下一些怀疑。」基本上，我们是在问：宇宙是不是按决定——决定论的方式运作的，这是个悬而未决的问题。你知道，我想是拉普拉斯说过：「你看，「如果你把宇宙中每一个粒子的位置和动量都告诉我，我就能推算出接下来的全部未来。」

拉普拉斯1814年提出「拉普拉斯妖」：若知所有粒子的位置和动量即可预言全部未来。量子力学的不确定性原理对此构成根本挑战。

grandiose /ˈgrændioʊs/ *adj.*
宏大的，浮夸的

deterministically /dɪˌtɜːrmɪˈnɪstɪkli/ *adv.*
决定论地（由初始条件唯一决定后续）

54:31

And this is leads to one of kind of the grand philosophical questions, which you know, we'll be investigating as part of this class, as well. Which is, you know, if the universe operates deterministically, if Newton's laws govern how my arm falls and how all the atoms in my body interact, where does free will creep into? How do I know I have control over these actions and it's not the fact that at the Big Bang there was a denser cluster of atoms over here and a less dense over here and things evolved according to deterministic laws, much like the formal systems we're playing with here.

这就引出了一个宏大的哲学问题，我们在这门课里也会去探讨。也就是说，如果宇宙是按决定论运行的，如果牛顿定律支配着我的手臂如何落下、我体内的所有原子如何相互作用，那自由意志从哪儿冒出来？我怎么知道我真的能控制这些动作，而不是因为在大爆炸的时候这边的原子团稍微密一些、那边稍微稀一些，然后一切就按照决定论的规律演化下去，就像我们这里在玩的这些形式系统一样。

决定论与自由意志之争：若牛顿定律决定一切原子运动，「我的选择」是否只是初始条件的必然展开？这是课程后半的哲学主线。

creep into *phr. v.*
悄悄潜入，不知不觉进入

55:10

So, this question you can really think of on two levels. One, can the universe be thought of as being modeled by a formal system, having forces and solving equations for the particles here, and it collides with another particle at this angle, they go off like this, and things like this, but it also I think likes to ask another question, which is version two, for those of you who are kind of matrix fans, to what extent is the universe a formal system proper in the sense? Is it a program, you know, running in the background of some hyperdimensional alien who's playing wow? And, you know, he's just running our universe as a simulation on his, you know, supercomputer cluster that he's got in his basement. who knows? I mean, if the universe is deterministic or he can just he's just coded up, you know, hacking away in Python all of our rules of our universe, and he said, all right, let's let this simulation go, and here we are in his computer having all these kind of dramatic interactions with people, et

所以这个问题其实可以分两个层面来看。第一，宇宙能不能被看作是由一个形式系统来建模的——有各种力，为这些粒子求解方程，它以某个角度和另一个粒子相撞，然后它们就这样弹开，诸如此类；但我觉得它还会引出另一个问题，也就是第二个版本，对于你们当中喜欢《黑客帝国》的人来说：宇宙在多大程度上本身就是一个严格意义上的形式系统？它是不是一个程序，在某个高维外星人的后台跑着，而那家伙正在玩《魔兽世界》？然后他只是把我们的宇宙当成一个模拟程序，跑在他放在地下室里的超级计算机集群上。谁知道呢？我是说，如果宇宙是决定论的，或者他只是把我们宇宙的所有规则用 Python 敲了出来，然后说，好，让这个模拟跑起来吧，于是我们就在他的计算机里，跟各种人上演这些戏剧性的互动，等等等等。

模拟假说：宇宙是否是高维存在运行的程序？哲学家博斯特罗姆2003年给出过正式论证。此处以《黑客帝国》作流行文化引子。

hyperdimensional /ˌhaɪpərdaɪˈmenʃənl/ *adj.*
超维的，高维的

hacking away *phr. v.*
埋头苦干（此处指不停地写代码）

56:19

Cetera, et cetera. and he's just kind of adjusted up what bug came up, et cetera. it's kind of interesting to think about. So, we've now really kind of hit home these five tools for thinking. and we're going to be revisiting all of these ideas throughout the entire book. And I and one of the things that Bock one of the things that Douglas Hofstadter does is he structures his book in its own kind of recursive fashion. And you know, I only gave you a few specific instances of where recursion shows up. And this represents kind of my bias. For me, I'm very much an art person and a math person, but I'm not so much of a music person. And I really encourage you guys to bring in different elements because GEB has such like a high-dimensional structure to it.

然后他就随手调一调，看看又冒出了什么 bug，等等。想想还挺有意思的。所以，我们现在算是真正把这五种思维工具讲透了。嗯，而且在整本书里我们还会不断回到这些概念。我想说的是，巴赫……呃，道格拉斯·侯世达做的一件事，就是他把自己这本书也编排成了一种递归的结构。而且你知道，我只举了几个递归出现的具体例子。这其实反映了我的个人偏好。对我来说，我是个偏艺术、偏数学的人，但对音乐就没那么在行。我真的鼓励大家把不同的元素带进来，因为《哥德尔、埃舍尔、巴赫》这本书有一种非常高维的结构。

bias /ˈbaɪəs/ *n.*
偏好，倾向性

hit home *phr.*
讲透，使人切实领会

12 巴赫赋格与对话结构

57:12

Everybody contributes their own slice to it. and one thing which I would hate to deny for deny you guys from is the music aspect of this book. Each one of Douglas Hofstadter's dialogues is actually structured and based upon a piece of Bach's music. If you listen to Bach's music and you read the dialogue, you might actually hint at some of the connection, some of the isomorphism that Hofstadter's alluding to. but first of all, you should know why he chose Bach, how recursion acts in music, and that's why I have this whole speaker set up here. So, allow me to play.

每个人都能贡献自己的那一个切面。嗯，而我最不愿意让你们错过的一点，就是这本书的音乐层面。道格拉斯·侯世达的每一段对话，其实都是依照巴赫的一首乐曲来构建的。如果你一边听巴赫的音乐一边读那段对话，你可能真的会隐约察觉到其中的某种联系、某种同构关系，侯世达所影射的东西。嗯，但首先，你们应该知道他为什么选了巴赫，递归在音乐中是如何体现的，这也是我在这儿架了这一整套音响设备的原因。那么，请允许我演奏一下。

GEB每章前的对话都以巴赫某首乐曲为结构模板（如《螃蟹卡农》对应回文式对话）——音乐、绘画、数学共享同构的形式结构。

alluding to /əˈluːdɪŋ/ *phr. v.*
暗指，间接提及

57:56

So, this is Bach's Little Fugue in G minor. just as a nice anecdote, who here has seen a beautiful mind? The movie? All right, so John Nash, the mathematician who went crazy, Princeton, etc. The story goes that he used to actually stalk around the halls of the math department smoking cigarettes and whistling this song constantly. And what were some of the things which you noticed about this piece? For those of you with good auditory abilities, what did you notice? There's certain patterns.

这是巴赫的《G小调小赋格》。嗯，说个有意思的小故事，在座有谁看过《美丽心灵》？那部电影？好，那么约翰·纳什，那位后来疯了的数学家，普林斯顿，等等。据说他以前经常在数学系的走廊里晃来晃去，抽着烟，不停地吹着这首曲子的口哨。那么，关于这首曲子，你们注意到了哪些地方？在座听觉比较灵敏的各位，你们注意到了什么？有某些模式。

《G小调小赋格》BWV 578。约翰·纳什：普林斯顿数学家、1994年诺贝尔经济学奖得主，患精神分裂症，《美丽心灵》(2001) 以其为原型。

anecdote /ˈænɪkdoʊt/ *n.*
轶事，趣闻

stalk /stɔːk/ *v.*
潜行，悄然游荡

auditory /ˈɔːdətɔːri/ *adj.*
听觉的

58:27

Okay, elaborate a little bit on these patterns. They're I don't know. I'm not a music person either. I've never played it so I don't know. Maybe it's like after a certain number it goes and repeats and Exactly. So, you heard it come in at a different tone, at a different volume. and you noticed it was the same theme. It's the same theme that he played stretched, inverted, backwards, on higher levels, on lower levels. So, GEB is actually very much structured like a fugue. Hofstadter lays out for us and what I did in this first lecture is I laid out the entire I'm laying out the entire book for you all in one go. So, that way you understand it when I play it stretched out, inverted, backwards, and at different volumes.

好，请稍微展开讲讲这些模式。他们……我不知道。我也不是搞音乐的人。我从来没弹过，所以我不知道。也许就是过了某个数之后它会……重复，没错。所以你听到它以不同的音调、不同的音量进来。嗯，而且你注意到那是同一个主题。就是他弹过的同一个主题，被拉长、倒置、逆行，在更高的声部、更低的声部上出现。所以《GEB》其实在结构上非常像一首赋格。侯世达为我们铺陈了这一点，而我在第一讲里做的，就是把整本书一次性地铺陈给大家。这样一来，当我把它拉长、倒置、逆行、用不同的音量演奏时，你们就能理解了。

赋格：同一主题在不同声部以移调、增值（拉长）、倒影、逆行等变换反复进入。讲者点明：本讲就是全书的「主题呈示部」。

fugue /fjuːg/ *n.*
赋格曲（同一主题在多声部中模仿追逐的复调音乐）

inverted /ɪnˈvɜːrtɪd/ *adj.*
倒置的，倒影的（音乐中指旋律上下翻转）

59:08

So, this is nice. You have a musical illustration, you have artistic illustrations of the ideas we're talking about. But we need to actually kind of settle into the book itself. So, Karen Kelleher and I or anyone else who's really excited about reading. Anybody really excited about volunteering for reading a dialogue? Anybody have the book with them right now? Oh, good job. would you like to read? You don't have to.

所以这挺好的。你们有了一个音乐上的例证，也有了我们所讨论的这些观念的艺术例证。但是我们其实得沉下心来进入这本书本身。所以，Karen Kelleher 和我，或者其他任何真的很想读一读的人。有谁特别想自告奋勇来读一段对话吗？现在有谁手边带着书吗？哦，很好。嗯，你愿意读吗？你不是非读不可。

59:46

You want to? Okay. So, we're going to spend the last kind of 15 minutes going through a dialogue. I actually have another copy. And So, I need two characters. One to be Achilles and one to be Tortoise. These are two characters we're going to meet in this dialogue. They're going to play a prominent role throughout the entire book. So, let's Does anyone else want to be Well, see I like the Tortoise, so I'd like to be the Tortoise, but someone else can be the Tortoise if they want to be.

你想读？好的。那我们就用最后大概 15 分钟来过一遍一段对话。我其实还有一本。嗯……还有，嗯……我需要两个角色。一个演阿基里斯，一个演乌龟。这是我们在这段对话里会遇到的两个角色。他们在整本书里都会扮演很重要的角色。那么，还有谁想演……你看，我喜欢乌龟，所以我想演乌龟，但是得有人……其他人如果想当乌龟也可以。

阿基里斯与乌龟源自芝诺的追龟悖论；刘易斯·卡罗尔1895年曾借这两个角色写逻辑对话，侯世达再借用为贯穿全书的主角。

60:20

Okay. So, we only have one soul that's brave enough to do it. All right. All righty. So, page 79. See, yeah, sorry.

好的。所以只有一位勇敢的同学愿意来。好吧。那好吧。那么，第 79 页。看，嗯，抱歉。

60:40

So, I'm going to give you some quick background on this dialogue. so, Hofstadter, like me, believes that it's important to introduce the idea of a of a topic conceptually first before you start really diving into it. So, he prefaces every chapter with a with a dialogue. And the dialogue is kind of a conceptual introduction to the ideas we're talking about. To go ahead and give you an idea of what this dialogue's based on, it's going to be the conflict of two mathematicians, Kurt Gödel and David Hilbert.

我先给大家简单介绍一下这段对话的背景。嗯，侯世达（Hofstadter）和我一样，认为在真正深入一个主题之前，先从概念上引入这个想法很重要。所以他在每一章前面都放了一段对话。这段对话算是我们要讨论的那些概念的一个引子。先让大家了解一下这段对话的基础，它讲的是两位数学家之间的冲突，嗯，库尔特·哥德尔（Kurt Gödel）和大卫·希尔伯特（David Hilbert）。

希尔伯特纲领（1920年代）：把全部数学形式化，并证明其一致且完备。哥德尔1931年证明此目标不可能——对话预演的正是这场冲突。

prefaces /ˈprefəsɪz/ *v.*
为…作序，在…之前加引子

61:14

David Hilbert believed that mathematics could be put into a formal system very rigorously, and it could also be proved to be consistent and complete. Those are two words which I'm going to have to define kind of at the end of this dialogue. But let's go ahead and start it off and try to work quickly through this. I'm going to ask that when you have the italics, you go ahead and read it if it's part of your section. So people have an idea what's going on in the in the book.

呃，大卫·希尔伯特相信数学可以被非常严格地纳入一个形式系统，而且还可以被证明是一致的和完备的。这两个词我得在这段对话结束时给大家定义一下。不过我们先开始吧，尽量快点过一遍。嗯，我要请大家注意，遇到斜体字的时候，如果那是你负责的部分，也请读出来。这样大家就能知道书里到底在发生什么。

一致性＝系统推不出矛盾；完备性＝所有真命题皆可证。哥德尔定理表明：包含算术的形式系统不可能两者兼得。

consistent /kənˈsɪstənt/ *adj.*
一致的（逻辑术语：推不出矛盾的）

61:50

All right, excellent. So we don't have really any time left. but I want to say one thing, it's a challenge. pay attention to Tortoise's quote on page 81 when she talks about acrostics. If you can find the two acrostics in this dialogue,

好，非常好。所以我们其实没剩多少时间了。嗯，但我想说一件事，这是个挑战。呃，注意乌龟在第 81 页的那句话，就是她谈到藏头诗（acrostics）的地方。如果你能在这段对话里找出那两个藏头诗，

acrostic（藏头文字）：取每行或每句首字母拼出隐藏信息。GEB 的对话中侯世达多处埋有这类文字游戏，与「层次隐藏意义」主题呼应。

acrostics /əˈkrɔːstɪks/ *n.*
藏头诗（首字母拼出隐藏信息的文字游戏）

第二部分 生词总表 · 复习用

词汇	音标	词性	释义	出现
abides by	—	phr. v.	遵守，恪守（规则）	23:29
acrostics	/əˈkrɔːstɪks/	n.	藏头诗（首字母拼出隐藏信息的文字游戏）	61:50
alluding to	/əˈluːdɪŋ/	phr. v.	暗指，间接提及	57:12
anecdote	/ˈænɪkdoʊt/	n.	轶事，趣闻	57:56
antinomies	/ænˈtɪnəmiz/	n.	二律背反，真正的逻辑悖论	18:20
auditory	/ˈɔːdətɔːri/	adj.	听觉的	57:56
axiom	/ˈæksiəm/	n.	公理，推理的起点	30:10
bias	/ˈbaɪəs/	n.	偏好，倾向性	56:19
coined	/kɔɪnd/	v.	创造（新词）；coin a term 造一个术语	13:06
condense	/kənˈdens/	v.	压缩，精简（内容）	6:21
configurations	/kənˌfɪgjəˈreɪʃnz/	n.	构型，排布方式	53:27
congruent	/ˈkɑːŋgruənt/	adj.	全等的（几何术语）	36:00
consistent	/kənˈsɪstənt/	adj.	一致的（逻辑术语：推不出矛盾的）	61:14
courtesy of	—	phr.	承蒙…提供，来自…（表来源致谢）	17:31
cranking through	—	phr. v.	机械地大量做完（乏味的工作）	38:05
creep into	—	phr. v.	悄悄潜入，不知不觉进入	54:31
derivation	/ˌderəˈveɪʃn/	n.	推导，推导过程（形式化证明）	35:38
derive	/dɪˈraɪv/	v.	推导出，导出	30:10
deterministically	/dɪˌtɜːrmɪˈnɪstɪkli/	adv.	决定论地（由初始条件唯一决定后续）	54:01
diagonalization	/daɪˌægənələˈzeɪʃn/	n.	对角线法（康托尔证明实数不可数的方法）	27:00
digression	/daɪˈgreʃn/	n.	离题，跑题的插话	13:06
elementary particles	—	phr.	基本粒子	52:58
entity	/ˈentəti/	n.	实体，独立存在物	1:44
falsidical	/fɔːlˈsɪdɪkl/	adj.	谬误性的（结论假、推理藏有错误的悖论）	18:20

feat	/fiːt/	n.	壮举，艰难的成就	0:00
fractals	/ˈfræktlz/	n.	分形（维数可为分数的自相似图形）	12:20
fugue	/fjuːg/	n.	赋格曲（同一主题在多声部中模仿追逐的复调音乐）	58:27
gasket	/ˈgæskɪt/	n.	垫片（Sierpinski gasket 谢尔宾斯基垫片/三角）	12:20
grandiose	/ˈgrændioʊs/	adj.	宏大的，浮夸的	54:01
hacking away	—	phr. v.	埋头苦干（此处指不停地写代码）	55:10
hit home	—	phr.	讲透，使人切实领会	56:19
homomorphism	/ˌhoʊməˈmɔːrfɪzəm/	n.	同态（允许丢失细节的结构映射）	10:11
hounds	/haʊndz/	v.	纠缠，穷追不舍地困扰	22:36
hyperdimensional	/ˌhaɪpərdaɪˈmenʃənl/	adj.	超维的，高维的	55:10
hyphens	/ˈhaɪfnz/	n.	连字符	45:07
incompleteness	/ˌɪnkəmˈpliːtnəs/	n.	不完备性（哥德尔不完备性定理）	4:49
induction	/ɪnˈdʌkʃn/	n.	归纳法（由个例推出一般结论）	43:00
inferring	/ɪnˈfɜːrɪŋ/	v.	推断，推知	47:27
intimately	/ˈɪntɪmətli/	adv.	密切地（intimately linked 紧密关联）	22:36
inverse	/ˌɪnˈvɜːrs/	n.	逆，逆映射	8:55
inverted	/ɪnˈvɜːrtɪd/	adj.	倒置的，倒影的（音乐中指旋律上下翻转）	58:27
isomorphism	/ˌaɪsəˈmɔːrfɪzəm/	n.	同构（结构间保持对应关系的映射）	4:49
law of the excluded middle	—	phr.	排中律（命题非真即假，无第三种可能）	21:40
logarithm	/ˈlɔːgərɪðəm/	n.	对数	15:50
mind-bending	/ˈmaɪndˌbendɪŋ/	adj.	烧脑的，令人费解的	13:06
mosaic	/moʊˈzeɪɪk/	n.	马赛克，镶嵌图案	13:06
negation	/nɪˈgeɪʃn/	n.	否定（逻辑术语）	21:40
obscure	/əbˈskjʊr/	v.	遮蔽，使模糊（此处作动词）	41:12
opiate	/ˈoʊpiət/	n.	鸦片剂；使人麻木之物（the opiate of the masses 人民的鸦片）	41:12
oppressing	/əˈpresɪŋ/	v.	压迫	40:21
overthrow	/ˌoʊvərˈθroʊ/	v.	推翻（政权）	40:21
pamphlets	/ˈpæmfləts/	n.	小册子，政论传单	40:21

paradoxical	/ˌpærəˈdɑːksɪkl/	adj.	自相矛盾的，悖论式的	19:43
perceive	/pərˈsiːv/	v.	察觉，看出	39:34
perceptive	/pərˈseptɪv/	adj.	敏锐的，观察力强的	15:08
pertinent	/ˈpɜːrtnənt/	adj.	密切相关的，切题的	5:42
prefaces	/ˈprefəsɪz/	v.	为…作序，在…之前加引子	60:40
primitives	/ˈprɪmətɪvz/	n.	基本元素，原始单元（逻辑/计算术语）	2:23
recursion	/rɪˈkɜːrʒən/	n.	递归（以自身定义自身的过程）	10:55
reduces to	—	phr. v.	归结为，简化为	33:33
renegade	/ˈrenɪgeɪd/	adj.	叛逆的，离经叛道的	37:35
reverse culture shock	—	phr.	反向文化冲击（回到本国后反而不适应）	0:00
rigorously	/ˈrɪgərəsli/	adv.	严格地，严密地	4:49
rules of inference	—	phr.	推理规则	36:00
scrambling	/ˈskræmblɪŋ/	v.	手忙脚乱地抢做，争抢	37:35
self-reference	/ˌselfˈrefərəns/	n.	自指，自我指涉	4:17
shorthand	/ˈʃɔːrthænd/	n.	速记，简写记法	47:27
shunting	/ˈʃʌntɪŋ/	n.	搬移，调动（symbol shunting 指机械地搬弄符号）	4:17
stalk	/stɔːk/	v.	潜行，悄然游荡	57:56
successor	/səkˈsesər/	n.	后继，后继数（皮亚诺算术术语）	34:48
syllabus	/ˈsɪləbəs/	n.	教学大纲，课程提纲	39:34
Tacit	/ˈtæsɪt/	adj.	心照不宣的，未言明的（tacit assumption 隐含假设）	52:58
tack	/tæk/	v.	附加（tack on 在末尾加上）	28:37
takes his sweet, sweet time	—	phr.	慢条斯理，不紧不慢地做（含轻微不耐烦意味）	27:00
tall order	—	phr.	艰巨的任务，过高的要求	5:42
typographical	/ˌtaɪpəˈgræfɪkl/	adj.	排版上的；此处指只涉及符号形状、不涉含义的	27:47
variant	/ˈveriənt/	n.	变体，变种	26:16
velocities	/vəˈlɑːsətiz/	n.	速度（物理术语，含方向）	53:27
veridical	/vəˈrɪdɪkl/	adj.	真实的，与事实相符的（veridical paradox 真实性悖论）	17:31
well-formed formula	—	phr.	合式公式（符合形式系统语法规则的符号串）	49:22

whirl	/wɜːrl/	v.	旋转，搅动	10:55